國立宜蘭大學 112 年度微積分競試 試題

※注意事項※

1. 考試時間為 100 分鐘(13:10-14:50)，考試開始 20 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機及任何通訊器材。
2. 本試題共計 22 題，總分為 102.6 分。
3. 各題答案請依題號填入答案卷上相對應題號的空格內，填錯格或填在格外者不予計分，字跡切勿潦草，答錯或未作答者，不給分亦不倒扣。
4. 請將您的班級、學號及姓名，用正楷填寫於答案卷上方的欄位內。
5. 考試結束時，請將答案卷繳回即可，本試題不必繳回。
6. 14:00 後才能提早交卷。

祝金榜題名!!!

1-8 題每題 4 分

1. Find the values of b and c, such that $f(x)$ is continuous on the entire real number line. $f(x) = \begin{cases} x+1 & 1 < x < 3 \\ x^2 + bx + c & |x-2| \geq 1 \end{cases}$ $(b, c) = ?$

2. Find the values of a and b>0 such that $\lim_{x \to 0} \frac{a - \cos(bx)}{x^2} = 2.$ $(a, b) = ?$

3. Evaluate $\lim_{x \to 1} \frac{(\ln x)^2}{x-1}$

4. Evaluate $\lim_{x \to \infty} (\ln x)^{2/x}$

5. Evaluate $\lim_{n \to \infty} \frac{1}{n} \left[\sqrt{\frac{2}{n}} + \sqrt{\frac{4}{n}} + \sqrt{\frac{6}{n}} + \cdots + \sqrt{\frac{2n}{n}} \right]$

6. Find the sum of the infinite series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$.

7. Find the sum of the series $\sum_{k=0}^{\infty} \frac{3 \cdot 2^k + 3^k}{5^k} = 4 + \frac{9}{5} + \frac{21}{25} + \frac{51}{125} + \cdots$.

8. Find the 4th Maclaurin polynomial for the function $f(x) = \cos x$.

9-16 題每題 5 分

9. Evaluate $\dfrac{d^{35}}{dx^{35}}(x \cdot \sin x)$

10. 試求出下列函數之微分 $y = \dfrac{\tan x - 1}{\csc x}$

11. 試求出下列函數之微分 $y = \log_2(3^x + x^4 + 5^6)$

12. 試求出下列函數之微分 $y = \ln\left(\log_{10} \sin x\right)$

13. Evaluate $\int_2^4 \frac{x}{\sqrt[3]{x^2-4}}dx$

14. Evaluate $\int_1^{\sqrt{3}} \frac{2x-3}{x^3+x}dx$

15. Find the first partial derivatives of the function. $f(x,y) = x^4 + 5xy^3$

16. Find the first partial derivatives of the function. $f(x,y) = ye^{xy}$

17-22 題每個答案 5.1 分

17. Find the gradient of $f(x, y) = x\cos y$.

18. Evaluate $\int_1^2 \int_0^2 y + 2xe^y \, dxdy$

19. Evaluate $\int_0^1 \int_0^x \cos(x^2) \, dydx$

20. Evaluate $\int_0^{\pi} \int_0^1 \int_0^{\sqrt{1-y^2}} y \sin x \, dzdydx$

21. Find the area bounded by the curves corresponding to the following functions in the first quadrant:

$y = f(x) = \cos\left(\frac{\pi x}{2}\right)$ and $y = g(x) = 1 - x^2$

22. Find the arc length of the curve corresponding to $y = f(x)$ on the x-interval of [0, ln 2], where

$$f(x) = \cosh x = \frac{e^x + e^{-x}}{2}$$