國立宜蘭大學 101 學年度微積分競試試題

※注意事項※

1. 考試時間為 80 分鐘，考試開始 10 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機、及任何通訊器材。
2. 試題共計 25 題，每題 4 分，試題答案請依題號填入答案卡，答錯或劃記多於一個選項者倒扣 1 分，倒扣到總分數零分為止，未作答者，不給分亦不倒扣。
3. 請用 2B 鉛筆在答案卡之「解答欄」內劃記。修正時應以橡皮擦拭，請勿在答案卡上使用修正液。作答範例：若第 1 題試題選項為(A)3 (B)5 (C)7 (D)9 (E)11，而正確的答案為選項(A)3 時，請在答案卷上劃記 (請實心填滿或大部分填滿) 如下圖：

國立宜蘭大學 101學年度微積分競試答案卷

系別:______________ 年級:______________

姓名:______________ 學號:______________

	A	B	C	D	E		A	B	C	D	E
1	■					14					
2						15					
3						16					
4						17					
5						18					
6						19					
7						20					
8						21					
9						22					
10						23					
11						24					
12						25					
13											

祝考試順利!!

1.Which statement is false?

(A) $e=\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^n$ (B) $e=\lim_{t\to 0}[1+t]^{1/t}$ (C) $\lim_{n\to\infty}\left[1+\frac{x}{n}\right]^n=e^x$ (D) $\lim_{t\to 0}[1+xt]^{1/xt}=e^x$ (E) $\lim_{t\to 0}[1+xt]^{1/t}=e^x$

2. Evaluate $\lim_{x\to 0}\frac{|x|}{x}=$

(A)-1 (B)0 (C)1 (D)2 (E)limit doesn't exist

3. Evaluate $\lim_{x\to 0}\frac{\sin 7x}{\sin 5x}=$

(A) $\frac{7}{5}$ (B) $\frac{5}{7}$ (C) 0 (D) $\frac{5}{3}$ (E) $\frac{3}{5}$

4. Let $f(u,v)=u^v$, where $u=\tan x$ and $v=\sin x$, please find $\frac{df}{dx}$ (Express the final answer in x only.)

(A) $\sin x\cdot\tan x$ (B) $\cos x\cdot\sec^2 x$ (C) $\sin x\cdot(\tan x)^{\sin x-1}\cdot\sec^2 x$ (D) $(\tan x)^{\sin x}\left[\frac{1}{\cos x}+\cos x\cdot\ln\tan x\right]$

(E) $(\tan x)^{\sin x}\cdot\cos x[1+\ln\tan x]$

5. Determine whether Rolle's Theorem can be applied to $f(x)=\frac{x^2-14}{x}$ on the closed interval $[-14,14]$. If Rolle's Theorem can be applied, find all values of c in the open interval $(-14,14)$ such that $f'(c)=0$

(A)c=1, c=10 (B)c=10 (C)c=4,c=1 (D)c=4 (E) Rolle's Theorem does not apply

6. Assume $y=f(x)$ is a continuous function, and satisfies $y=x^{x+y}$,please find $y'(1)$

(A)-2(B) -1(C) 0(D) 1(E)2

7. Evaluate $\int_0^1 x^5 e^{x^3}dx$

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E)1

8. If $f(x)$ is a continuous function defined in R, and $\int_0^{x^2} f(t)dt=x\sin x$, then $f(1)=?$

(A)1 (B) $\frac{-\pi}{2}$ (C) 0 (D) $\frac{\pi}{2}$ (E) -2

9. Assume $F(x)=\int_0^{x^2}\sin\theta^2 d\theta$, please find the $F'(x)$.

(A) $2x\sin x^4$ (B) $2x\sin x^2$ (C) $2x\cos x^4$ (D) $2x\cos x^2$ (E) $\sin x^4\cdot\cos x^4$

10. Evaluate $\int_0^1 \frac{x^3-1}{\ln x}dx = ?$

(A) ln 3 (B) 2 ln 3 (C)2ln 2 (D) 3ln 2 (E) $3\ln 3$

11. Evaluate $\int \csc x dx = ?+C$ $\left(C \text{ is constant} \right)$

(A) $\ln\left|\frac{\sin x}{1+\cos x}\right|$ (B) $\ln\left|\csc x + \cot x\right|$ (C) $\sec x$ (D) $-\sec x$ (E) no solution

12. Which of the following step begin worse?

$\int_{-\infty}^{\infty} x^3 dx \underset{(A)}{=} \lim_{t\to\infty}\int_{-t}^{t} x^3 dx \underset{(B)}{=} \lim_{t\to\infty}\frac{1}{4}x^4\Big|_{-t}^{t} \underset{(C)}{=} 0$ (D)is correct. (E)cannot identify

13. Evaluate $\int \frac{1}{a^2+x^2}dx = ?+C$ $\left(C \text{ is constant} \right)$

(A) $\frac{1}{a}\cdot\tan\frac{x}{a}$ (B) $sec^{-1}x$ (C) $\frac{1}{a}\cdot\sin^{-1}x$ (D) $\frac{1}{a}\cdot\cos^{-1}x$ (E) $\frac{1}{a}\cdot\tan^{-1}\frac{x}{a}$

14. Evaluate $\int \frac{1}{a^4+x^4}dx = ?+C$ $\left(C \text{ is constant} \right)$

(A) $\frac{1}{2a^3}\left[\ln|a+x| + \ln|a-x| + 2\tan^{-1}\frac{x}{a}\right]$ (B) $\frac{1}{4a^3}\left[\ln|a+x| - \ln|a-x| + 2\tan^{-1}\frac{x}{a}\right]$

(C) $\frac{3}{4a^3}\left[\ln|a+x| - \ln|a-x| + \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$ (D) $\frac{x}{4a^3}\left[\ln|a+x| - \ln|a-x| + \frac{x}{a}\tan^{-1}\frac{x}{a}\right]$

(E) $\frac{1}{4a^3}\left[\ln|2a+x| + \ln|a-x| + \tan^{-1}\frac{x}{a}\right]$

15. If $f(x)=\sqrt{x^3+x+6}$, find $(f^{-1})'(4)$

(A) $\frac{3}{7}$ (B) $\frac{4}{9}$ (C) $\frac{5}{11}$ (D) $\frac{8}{13}$ (E) $\frac{6}{15}$

16. Evaluate $\int_1^5 \frac{\ln x}{x^3}dx = ?$

(A) $\frac{1872}{625}$ (B) $\frac{-1}{3}\left[\frac{2\ln 7+1}{50}-\frac{1}{2}\right]$ (C) $\frac{24-2\ln 5}{100}$ (D) $\frac{-1}{2}\left[\frac{2\ln 5+1}{50}-\frac{1}{4}\right]$ (E) $\frac{1}{2}\left[\frac{2\ln 5+1}{50}-\frac{1}{2}\right]$

17. Considering a surface: $f(x,y)=x^2+3xy+y^2+2$, please find the equation of <u>tangent plane</u> at point $(1,2,1)$.

(A) $7x+5y-3z=18$ (B) $8x+7y-z=21$ (C) $x+2y+z=15$ (D) $5x+5y+5z=25$ (E) $x+2y-3z=4$

18. If we know $\sinh x = \dfrac{e^{x} - e^{-x}}{2}$, $\cosh x = \dfrac{e^{x} + e^{-x}}{2}$, please evaluate $\int_{0}^{\ln 2} \tanh x dx = ?$

(A) $\ln\left(\dfrac{5}{2}\right)$ (B) $\ln\left(\dfrac{5}{3}\right)$ (C) $\ln\left(\dfrac{5}{4}\right)$ (D) $\ln\left(\dfrac{5}{6}\right)$ (E) $\ln\left(\dfrac{5}{7}\right)$

19.Evaluate $\int \dfrac{\cosh x}{\sqrt{9 - \sinh^{2} x}} dx = ?$

(A) $\arcsin\left(\dfrac{e^{x} - e^{-x}}{2}\right) + C$ (B) $\arcsin\left(\dfrac{e^{x} - e^{-x}}{6}\right) + C$ (C) $\arccos\left(\dfrac{e^{x} - e^{-x}}{6}\right) + C$ (D) $\arcsin\left(\dfrac{e^{x} + e^{-x}}{6}\right) + C$

(E) $\arcsin\left(\dfrac{e^{x} + e^{-x}}{2}\right) + C$

20. Find the curvature κ of the ellipse $\dfrac{x^{2}}{4} + \dfrac{y^{2}}{1} = 1$ at $x=0$.

(A)0 (B) 2 (C)$\dfrac{1}{2}$ (D) 4 (E)$\dfrac{1}{4}$

21. Use the definition of Taylor series to find the Taylor series for function $f(x) = \ln(x^{2} + 1)$, centered at $c = 0$

(A)$\sum_{n=0}^{\infty} \dfrac{(-1)^{n} x^{n+2}}{2n+1}$ (B)$\sum_{n=0}^{\infty} \dfrac{(-1)^{n} x^{2n+2}}{n+1}$ (C)$\sum_{n=0}^{\infty} \dfrac{(-1)^{n} x^{2n+3}}{2n+1}$ (D)$\sum_{n=0}^{\infty} \dfrac{(-1)^{n+1} x^{2n+2}}{3n+1}$ (E)$\sum_{n=0}^{\infty} \dfrac{(-1)^{n} x^{2n+3}}{n+2}$

22. Use spherical coordinates to find the volume of the solid. Solid inside $x^{2} + y^{2} + z^{2} = 9$,outside $z = \sqrt{x^{2} + y^{2}}$,and above the xy-plane.

(A)$9\pi\sqrt{2}$ (B)$9\pi\sqrt{5}$ (C)$7\pi\sqrt{3}$ (D)$7\pi\sqrt{5}$ (E)$7\pi\sqrt{11}$

23. Assume that Plane A is parallel to Plane B and they are flying at the same height. If Plane B, 200 kilometers away from Plane A in the southwest, is flying towards the south at 40 kilometers per hour, and Plane B is flying towards the east at 20 kilometers per hour, when will it occur that the distance between Plane A and Plane B is the shortest?

(A)$3\sqrt{3}$ (B)$2\sqrt{3}$ (C)$2\sqrt{5}$ (D)$3\sqrt{5}$ (E)$3\sqrt{2}$

24. According to question 23, what is the minimum in kilometers?

(A)$17\sqrt{2}$ (B)$15\sqrt{3}$ (C)$20\sqrt{10}$ (D)$25\sqrt{2}$ (E)$18\sqrt{10}$

25.Evaluate $\int \dfrac{-1}{4x - x^{2}} dx = ?$

(A) $\dfrac{1}{2}\ln\left|\dfrac{x-4}{x}\right| + C$ (B) $\dfrac{1}{3}\ln\left|\dfrac{x-4}{x}\right| + C$ (C) $\dfrac{1}{5}\ln\left|\dfrac{x-16}{x}\right| + C$ (D) $\dfrac{1}{4}\ln\left|\dfrac{x-4}{x}\right| + C$ (E) $\dfrac{1}{3}\ln\left|\dfrac{x-8}{2x}\right| + C$