國立宜蘭大學 101 學年度第二學期微積分競試試題

※注意事項※

1. 考試時間為 100 分鐘，考試開始 10 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機、及任何通訊器材(參考數值請參照 Appendix)。
2. 試題共計 25 題，每題 4 分，試題答案請依題號填入答案卡，答錯或劃記多於一個選項者倒扣 1 分，倒扣到總分數零分為止，未作答者，不給分亦不倒扣。
3. 請用 2B 鉛筆在答案卡之「解答欄」內劃記。修正時應以橡皮擦拭，請勿在答案卡上使用修正液。作答範例：若第 1 題試題選項為(A)3 (B)5 (C)7 (D)9 (E)11，而正確的答案為選項(A)3 時，請在答案卷上劃記 (請實心填滿或大部分填滿) 如下圖：

國立宜蘭大學101學年度微積分競試答案卷

系別:_____年級:_____

姓名:_____學號:_____

	A	B	C	D	E		A	B	C	D	E
1	■					14					
2						15					
3						16					
4						17					
5						18					
6						19					
7						20					
8						21					
9						22					
10						23					
11						24					
12						25					
13											

祝考試順利!!

1. Evaluate $\lim_{n\to\infty}\left(2^n+4^n\right)^{\frac{1}{n}}=?\quad n\in \mathrm{N}$

(A) 0 (B)1 (C) 2 (D) 4 (E) Does not exist

2. Evaluate $\lim_{x\to 4}\dfrac{\sqrt{x+12}-x}{x^4-4x^3}$

(A) $-\dfrac{7}{512}$ (B) $-\dfrac{7}{412}$ (C) $\dfrac{7}{412}$ (D) $-\dfrac{7}{502}$ (E) $\dfrac{7}{532}$

3. Evaluate $\lim_{x\to\frac{\pi}{4}}\left[\tan 2x\cdot\tan\left(\dfrac{\pi}{4}-x\right)\right]$

(A)0 (B) $\dfrac{1}{2}$ (C)1 (D)2 (E) Does not exist

4. Evaluate $\dfrac{d}{dx}\left[\log_{\sqrt{3}}\tan^{-1}(1+x)\right]$

(A) $\dfrac{2}{\ln 3\cdot\tan^{-1}(1+x)\cdot\left(x^2+2x+2\right)}$ (B) $\dfrac{2}{\ln 3\cdot\tan^{-1}(1+x)\cdot\left(x^2+2x+2\right)^2}$ (C) $\dfrac{3}{\ln 3\cdot\tan^{-1}(1+x)\cdot\left(x^2+2x+2\right)}$

(D) $\dfrac{2}{\ln 3\cdot\tan^{-1}(1-x)\cdot\left(x^2+2x+2\right)}$ (E) $\dfrac{\sqrt{3}}{\ln 3\cdot\tan^{-1}(1+x)\cdot\left(x^2+2x+2\right)}$

5. If $y=2^{\tan^{-1}x}+(\ln x)^{\sqrt{x}}$, please find $\dfrac{dy}{dx}$.

(A) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1+x^2}+(\ln x)^{\sqrt{x}}\left[\dfrac{\ln(\ln x)}{\sqrt{x}}+\dfrac{\sqrt{x}}{x\ln x}\right]$ (B) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1-x^2}+(\ln x)^{\sqrt{x}}\left[\dfrac{\ln(\ln x)}{\sqrt{x}}+\dfrac{\sqrt{x}}{x\ln x}\right]$

(C) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1+x^2}+(\ln x)^{\sqrt{x}}\left[\dfrac{\ln(\ln x)}{2\sqrt{x}}+\dfrac{\sqrt{x}}{x\ln x}\right]$ (D) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1-x^2}+(\ln x)^{\sqrt{x}}\left[\dfrac{\ln(\ln x)}{2\sqrt{x}}+\dfrac{\sqrt{x}}{x\ln x}\right]$

(E) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1-x}+(\ln x)^{\sqrt{x}}\left[\dfrac{\ln(\ln x)}{\sqrt{x}}+\dfrac{\sqrt{x}}{x\ln x}\right]$

6. Considering a function $\sin(xy)=x^2\cos y$, please find the equation of normal line at point $\left(2,\dfrac{\pi}{2}\right)$

(A) $y-\dfrac{\pi}{2}=\dfrac{\pi}{4}(x-2)$ (B) $y-\dfrac{\pi}{2}=-\dfrac{\pi}{4}(x-2)$ (C) $y-\dfrac{\pi}{2}=\dfrac{4}{\pi}(x-2)$ (D) $y-\dfrac{\pi}{2}=-\dfrac{4}{\pi}(x-2)$

(E) $y-\dfrac{\pi}{2}=\dfrac{\pi}{2}(x-2)$

7. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius is 5. (Fig.7)

(A) $\frac{3125\pi}{27}$ (B) $\frac{1500\pi}{9}$ (C) $\frac{2500\pi}{9}$ (D) $\frac{3750\pi}{27}$ (E) $\frac{4000\pi}{81}$

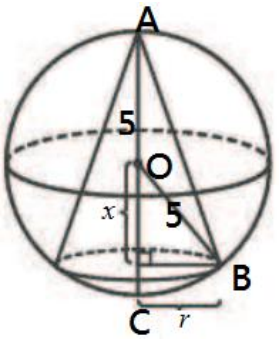

Fig.7

8. If $f(x)$ is continuous on [0 , 2] and differentiable in (0 , 2). Suppose that $f(0)=2$ and $1<f'(x)<2$ for all x in (0 , 2). Find a possible value of $f(2)$

(A)3 (B)4 (C)5 (D)6 (E) 7

【Problem Sets 9~10】Continuous Compounding.

設銀行年利率為 r，每年複利 n 次，本金為 P，

則 t 年後的本利和為 $P_t = P\left(1+\frac{r}{n}\right)^{nt}$

若每年複利一次，則 $\Rightarrow P_t = P(1+r)^n$

若每月複利一次，則 $\Rightarrow P_t = P\left(1+\frac{r}{12}\right)^{12t}$

若每週複利一次，則 $\Rightarrow P_t = P\left(1+\frac{r}{52}\right)^{52t}$

若每日複利一次，則 $\Rightarrow P_t = P\left(1+\frac{r}{365}\right)^{365t}$

若每個時刻都在複利，則稱為連續複利，此時 $\Rightarrow P_t = \lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt}$

9.設彥華在年初向銀行存了 20,000，年利率為 5%，採連續複利計算，則 10 年後彥華可領回多少錢?

(A)54366 (B)45636 (C)64356 (D)35466 (E) 66345

10.若彥華想等錢加倍後再領回，至少需要等幾年?

(A)12 (B)14 (C)16 (D)17 (E) 18

11. If $f(x)$ is a continuous function defined in R, and $\int_0^{x^2} f(t)dt = x\sin\pi x$, please find $f(9)$.

(A) -3π (B) $\frac{2\pi}{3}$ (C) $-\frac{2\pi}{3}$ (D) 3π (E) $-\frac{1}{2}\pi$

12. Evaluate $\int \cos\sqrt{x}dx$

(A) $\frac{\cos\sqrt{x}+\sin\sqrt{x}}{\sqrt{x}}+C$ (B) $\sqrt{x}\left(\cos\sqrt{x}+\sin\sqrt{x}\right)+C$ (C) $2\left(\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}\right)+C$

(D) $\sqrt{x}\sin\sqrt{x}+\frac{\cos\sqrt{x}}{\sqrt{x}}+C$ (E) $\frac{\sin\sqrt{x}}{\sqrt{x}}+\sqrt{x}\cos\sqrt{x}+C$

13. Evaluate $\int \frac{1}{4+4x^2+x^4}dx$

(A) $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{x}{x^2+2}+C$ (B) $\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{x}{4\left(x^2+2\right)}+C$ (C) $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{x}{2\left(x^2+2\right)}+C$

(D) $\frac{\sqrt{2}}{8}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{x}{\left(x^2+2\right)}+C$ (E) $\frac{\sqrt{2}}{8}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{x}{4\left(x^2+2\right)}+C$

14. Evaluate $\int \frac{\sec x}{\ln\left|\sec x+\tan x\right|}dx=$

(A) $\tan x+C$ (B) $\frac{1}{2}\sec x+C$ (C) $\ln\left|\sec x+\tan x\right|+C$ (D) $\ln\left|\ln\left|\sec x+\tan x\right|\right|+C$

(E) $\frac{1}{2}\sec x\tan x+\frac{1}{2}\ln\left|\sec x+\tan x\right|C$

15.Evaluate $\int_0^1 \frac{x+1}{x^2+1}dx=$

(A) $1-\ln 2$ (B) $\frac{4\ln 2-\pi}{6}$ (C) $\frac{\pi-\ln 2}{8}$ (D) $\frac{2\ln 2+\pi}{4}$ (E) $\frac{2\ln 2-\pi}{2}$

【Problem Sets 16~17】

16. Evaluate $\int \frac{\tan^{-1}x}{x^2}dx$

(A) $-\frac{\tan^{-1}x}{x}+\frac{1}{2}\ln\left(\frac{x^2}{x^2+1}\right)+C$ (B) $-\frac{\sec^{-1}x}{x}-\frac{1}{2}\ln\left(\frac{x}{x^2+1}\right)+C$ (C) $-\frac{\tan^{-1}x}{x^2}+\frac{1}{2}\ln\left(\frac{x^2+1}{x^2}\right)+C$

(D) $\frac{\sec^{-1}x}{x}\cdot\frac{1}{2}\ln\left(\frac{x^2}{x^2+1}\right)+C$ (E) $\frac{\tan^{-1}x}{x}+\frac{1}{2}\ln\left(\frac{x^2+1}{x^2}\right)+C$

17. Evaluate $\int_1^\infty \frac{\tan^{-1}x}{x^2}dx$

(A) $\frac{\pi}{2}+\frac{1}{4}\ln 2$ (B) $\frac{\pi}{4}-\frac{1}{2}\ln 5$ (C) $\frac{2\pi}{3}+\frac{1}{5}\ln 5$

(D) $\frac{3\pi}{4}-\ln 2$ (E) $\frac{\pi}{4}+\frac{1}{2}\ln 2$

18. Evaluate $\int \ln\left(x^2+2\right)dx$

(A) $x\cdot\ln\left(x^2+2\right)-2x+2\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$ (B) $\left(x^2+2\right)\cdot\ln x-2x^2+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$

(C) $x\cdot\ln\left(x^2+2\right)+2x+5\sqrt{2}\sec^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$ (D) $x^2\cdot\ln x^2-2x+2\sqrt{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$

(E) $\ln\left(x^2+2\right)-2x^2-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{x}\right)+C$

19. Please find the area of the region bounded by $y=0$、$x=0$、$x=1$ and the curve represented by parametric:

$x=\ln t$, $y=\frac{t+t^{-1}}{2}$.

(A) $\cosh 1$ (B) $\sinh 1$ (C) $\coth 1$ (D) $\tanh 1$ (E) $\cosh 2$

20. The region enclosed According to question19, is rotated about the line $y=0$. Please find the volume of the resulting solid.

(A) $\pi\left[\frac{1}{4}+\frac{1}{4}\sinh 2\right]$ (B) $\pi\left[\frac{1}{4}+\frac{1}{2}\sinh 2\right]$ (C) $\pi\left[\frac{1}{2}+\frac{1}{2}\sinh 2\right]$ (D) $\pi\left[\frac{1}{2}+\frac{1}{4}\sinh 2\right]$

(E) $\pi\left[\frac{1}{4}-\frac{1}{2}\sinh 2\right]$

21. Define that $erf(\eta)=\frac{2}{\sqrt{\pi}}\int_0^{\eta}e^{-\eta^2}d\eta$, where $\eta=\frac{z}{\sqrt{4kt}}$, k and t both are constant, please evaluate

$\frac{d}{dz}\left(erf(\eta)\right)=?$

(A) $\frac{e^{-\eta^2}}{\sqrt{4\pi kt}}$ (B) $\frac{2e^{-\eta^2}}{\sqrt{\pi kt}}$ (C) $\frac{e^{-\eta^2}}{\sqrt{\pi kt}}$ (D) $\frac{4e^{-\eta^2}}{\sqrt{\pi kt}}$ (E) can not solve.

22. Evaluate $\int_0^{\ln 2}2e^{-x}\cosh x dx$

(A) $\frac{1}{2}+\ln 2$ (B) $\frac{3}{8}-\ln 2$ (C) $\frac{1}{4}+\ln 2$ (D) $\frac{3}{4}+\ln 2$ (E) $\frac{3}{8}+\ln 2$

23. Evaluate $\int_0^{\infty}e^{-nt}\cdot\cos\alpha t\cdot dt$ $\left(n,\alpha \text{ are constant}\right)$

(A) $\frac{n^2}{n^2+\alpha^2}$ (B) $\frac{n}{n^2+\alpha^2}$ (C) $\frac{\alpha^2}{n^2+\alpha^2}$ (D) $\frac{\alpha}{n^2+\alpha^2}$ (E) $\frac{\beta}{n^2+\alpha^2}$

24. Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plan region about the *y*-axis.

$y=\begin{cases}\dfrac{\sin x}{x}, x>0\\ 1, x=0\end{cases}$，$y=0$，$x=0$，$x=\pi$

(A) 8π (B) 7π (C) 6π (D) 5π (E) 4π

25. Evaluate $\int_{-7}^{7}\dfrac{\sin(5x)}{\cos x\sqrt{1+x^2+x^4+x^6}}\cdot dx$

(A)7 (B) 0 (C)0.49 (D)-7 (E)14

Appendix

1. $e\approx 2.7183$

2. $\begin{cases}\ln 2\approx 0.6931\\ \ln 3\approx 1.0986\\ \ln 4\approx 1.3863\\ \ln 5\approx 1.6094\end{cases}$ 3. $\begin{cases}\log 2\approx 0.3010\\ \log 3\approx 0.4771\\ \log 4\approx 0.6021\\ \log 5\approx 0.6990\end{cases}$

4. $\sinh x=\dfrac{e^x-e^{-x}}{2}$ 5. $\cosh x=\dfrac{e^x+e^{-x}}{2}$ 6. $\tanh x=\dfrac{\sinh x}{\cosh x}$

7. If $\sin^{-1}x=y\Rightarrow \sin y=x$

8. $\pi\approx 3.1415926$

9. cone:錐體

10. compounding:複利

11. represented:代表

12. parametric:參數