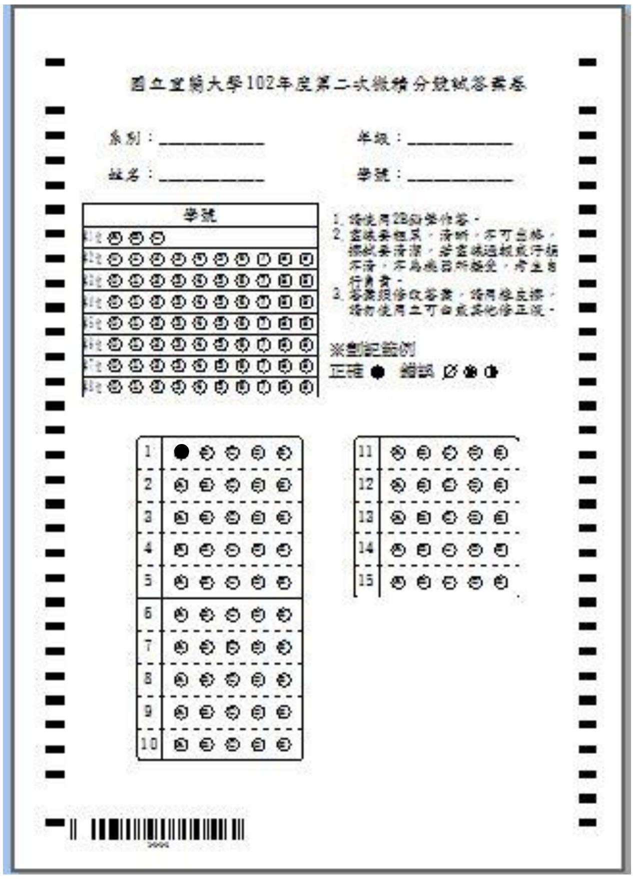
國立宜蘭大學 102 學年度第一學期微積分競試試題

※注意事項※

1. 考試時間為 100 分鐘，考試開始 10 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機、及任何通訊器材(參考數值請參照 Appendix)。
2. 試題共分為選擇題及非選題二部分，第一部份為單選題，每題 4 分，試題答案請依照題號填入答案卡，答錯或劃記多於一個選項者倒扣 1 分，倒扣到總分數零分為止，未作答者，不給分亦不倒扣；第二部分為非選題，請使用黑色或藍色原子筆作答。
3. 請用 2B 鉛筆在答案卡之「解答欄」內劃記。修正時應以橡皮擦拭，請勿在答案卡上使用修正液。作答範例：若第 1 題試題選項為(A)3 (B)5 (C)7 (D)9 (E)11，而正確的答案為選項(A)3 時，請在答案卷上劃記 (請實心填滿或大部分填滿) 如下圖：



祝考試順利!!

第一部分:單選題15題，每題答對得4分，答錯或選項多於一個者倒扣一分，未作答不予計分。

1. Evaluate $\lim\limits_{x\to\infty}\dfrac{x-\cos x}{x^3\tan^2\left(\dfrac{2}{x}\right)}$ = ?

(A) -1 (B) 0 (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{4}$ (E) 1

2. Let $f(x)=\dfrac{e^{9x^2+100}\sqrt{3x^2-1}}{x^3+5x-3}$, find $f'(x)$=?

(A) $e^{9x^2+100}\cdot\left(\dfrac{6x}{3x^2-1}-\dfrac{3x^2+5}{x^3+5x-3}\right)$ (B) $e^{9x^2+100}\cdot\left(4x+\dfrac{4x}{3x^2-1}+\dfrac{x^2+4}{3x^3+5x-3}\right)$

(C) $\dfrac{e^{9x^2+100}\sqrt{3x^2-1}}{x^3+5x-3}\left(18x+\dfrac{3x}{3x^2-1}-\dfrac{3x^2+5}{x^3+5x-3}\right)$ (D) $\dfrac{e^{9x^2+100}\cdot(3x^2-1)^{\frac{3}{2}}}{(x^3+5x-3)^2}\left(\dfrac{5x}{3x^2-1}+\dfrac{8x}{5x^2+4}\right)$

(E) $\dfrac{e^{9x^2+100}\cdot(3x^2-1)^2}{2(2x^3+5x-3)}\left(6x+\dfrac{3x^2-7}{4x^3+5x-1}\right)$

3. Suppose that $f(0)=-3$ and $f'(x)\le 5$ for all values of x. How large can $f(2)$ possibly be ?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

4. Evaluate $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-}(\tan x)^{\cos x}=?$

(A) 0 (B) 1 (C) $\dfrac{\sqrt{2}}{2}$ (D) $\dfrac{1}{2}$ (E) not exist

5. Evaluate $\lim\limits_{x\to 3}\dfrac{x}{x-3}\displaystyle\int_3^x\frac{\sin t}{t}dt=?$

(A) $\sin 3$ (B) $\sin 9$ (C) $\cos 3$ (D) $\cos 9$ (E) $\tan 9$

6. Evaluate $\int x\sin^{-1}x\,dx=?$

(A) $\dfrac{1}{2}\left(x^2\cdot\sin^{-1}x+\dfrac{1}{2}x\sqrt{1-x^2}\right)+c$ (B) $\dfrac{1}{4}\left(\sin^{-1}x-2x\sin^{-1}x+\sqrt{1-x^2}\right)+c$

(C) $\dfrac{1}{8}\left(x^2\sin^{-1}x-\dfrac{1}{2}\sin^{-1}x\cdot 3x+x^2\sqrt{1-x^2}\right)+c$ (D) $\dfrac{3}{4}\left(x\sin^{-1}x-2x\sin^{-1}x+x\sqrt{1-x^2}\right)+c$

(E) $\dfrac{1}{2}x^2\sin^{-1}x-\dfrac{1}{4}x\cdot\sin^{-1}x+2x\sqrt{1-x^2}+c$

7. Evaluate $\int_2^{\infty}\frac{2x^3+2x^2+x-1}{x^2(x-1)(x^2+1)}dx=?$

(A) $\frac{\pi}{4}+\ln 5-\tan^{-1}4$ (B) $\frac{\pi}{2}+4-\ln 2+\sin^{-1}3$ (C) $\frac{\sqrt{2}}{2}+\frac{1}{2}+\ln 4-\sin^{-1}2$

(D) $\frac{\sqrt{2}}{2}+1+\ln 5-\sin^{-1}4$ (E) $\frac{\pi}{2}+\frac{1}{2}+\ln 5-\tan^{-1}2$

8. Which of the following series is convergent ?

(A) $\sum_{n=1}^{\infty} n\ln\left(1+\frac{1}{n}\right)$ (B) $\sum_{n=1}^{\infty}\frac{n!}{n3^n}$ (C) $\sum_{n=1}^{\infty}\frac{1}{1+\ln n}$

(D) $\sum_{n=1}^{\infty}\sin\frac{1}{n}$ (E) $\sum_{n=1}^{\infty}2^n\sin\left(\frac{\pi}{3^n}\right)$

9. Evaluate $\lim_{(x,y)\to(0,0)}\frac{xy^2}{x^4+y^2}=?$

(A) -1 (B) 0 (C) 1 (D) $\frac{1}{2}$ (E) Does not exist

10. Let $f(x,y,z)=x^{y^z}$, find $\frac{\partial f}{\partial z}$ =?

(A) $x^{y^z}\cdot y\ln x\cdot y^z\cdot\ln y$ (B) $x^{y^z}\cdot\ln x\cdot z\cdot y^{z-1}$

(C) $x^{y^z}\cdot\ln x\cdot y^z\cdot\ln y$ (D) $x^{y^z}\cdot\ln x\cdot z\cdot y^{z-1}$ (E) $x^{y^z}\cdot y\ln x\cdot z\cdot y^{z-1}$

11. Find the value of $\frac{\partial x}{\partial z}$ at point (1,-1,3) if the equation $xz+y\ln x-x^2+4=0$.

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{8}$ (E) 1

12. Consider $f(x,y,z)=x^2y+y^3z+xz^3$ at the point (2,1,-1) .Find the maximum rate of change of $f(x,y,z)$ =?

(A) $\sqrt{31}$ (B) $\sqrt{37}$ (C) $\sqrt{47}$ (D) $\sqrt{59}$ (E) $\sqrt{61}$

13. If $x^2+y^2+z^2=48$, find the maximum of xyz =?

(A) 64 (B) 88 (C) 100 (D) 120 (E) 168

14. Evaluate the integral $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dydx = ?$

(A) $e^2 - 1$ (B) $\frac{3}{4}\left(e^2 - 2\right)$ (C) $e^6 - 4$ (D) $\frac{1}{6}\left(e^4 - 2\right)$ (E) $\frac{1}{4}\left(e^8 - 1\right)$

15. Evaluate the integral $\iint_R e^{\frac{x+y}{x-y}} dxdy$, where R is the trapezoidal region with vertices (1,0) , (2,0) , (0,-2) and (0,-1) .

(A) $\frac{1}{2}\left(e - 1\right)$ (B) $\frac{3}{4}\left(e + e^{-1}\right)$ (C) $\frac{3}{2}\left(e^2 - e\right)$ (D) $\frac{2}{3}\left(e^2 - e^{-1}\right)$ (E) $\frac{1}{5}\left(e^2 - e - 1\right)$

第二部分:非選題兩題，每題各兩小題，共計40分，請將答案用藍黑色原子筆寫在答案紙上。

1. Given a function f which satisfies the differential equation

$$xf''(x)+3x[f'(x)]^2=1-e^{-x}$$

for all real x .(Do not attempt to solve this differential equation).

(a) If f has an extremum at a point $c\neq 0$, show that this extremum is a minimum.

(b) If f has an extremum at 0, is it a maximum or a minimum? Justify your conclusion.

2. (a) Let u be a nonzero solution of the second-order equation

$$y''+P(x)y'+Q(x)y=0$$

Show that the substitution $y=uv$ converts the equation

$$y''+P(x)y'+Q(x)y=R(x)$$

into a first-order linear equation for v' .

(b) Obtain a nonzero solution of the equation $y''-4y'+x^2(y'-4y)=0$ by inspection and use the method of part (a) to find a solution of

$$y''-4y'+x^2(y'-4y)=2xe^{-x^3/3}$$

such that $y=0$ and $y'=0$ when $x=0$.