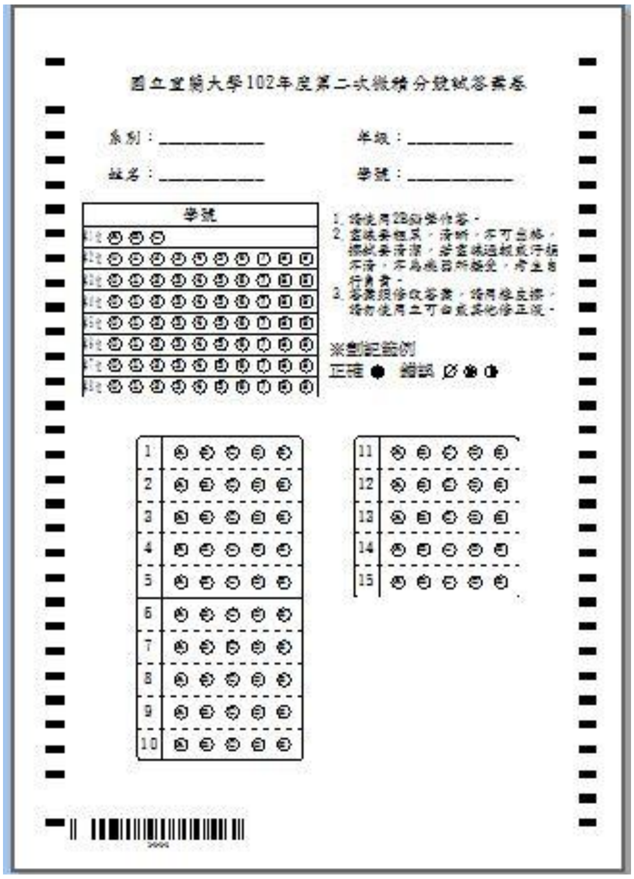
國立宜蘭大學 103 年度第一次微積分競試試題

※注意事項※

1. 考試時間為 100 分鐘，考試開始 10 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機、及任何通訊器材。
2. 選擇部分試題共計 20 題，每題 4 分，試題答案請依題號填入答案卡，答錯或劃記多於一個選項者倒扣 1 分，倒扣到總分數零分為止，未作答者，不給分亦不倒扣。
3. 請用 2B 鉛筆在答案卡之「解答欄」內劃記。修正時應以橡皮擦拭，請勿在答案卡上使用修正液。作答範例：若第 1 題試題選項為(A)3 (B)5 (C)7 (D)9 (E)11，而正確的答案為選項(A)3 時，請在答案卷上劃記 (請實心填滿或大部分填滿) 如下圖：



祝考試順利!!

第一部分:單選題 20 題，每題答對得 4 分，答錯或選項多於一個者倒扣一分，未作答者不予計分。

1. Evaluate $\lim_{x\to\infty}\frac{3^x+5^x}{3^{x+1}+5^{x+3}}=?$

(A)$\frac{1}{5}$(B)$\frac{1}{25}$(C)$\frac{1}{125}$(D)$\frac{1}{625}$(E)Does not exist.

2. Evaluate $\lim_{x\to1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$=?

(A)$\frac{2}{3}$ (B)1 (C)$\frac{3}{2}$ (D)0 (E)∞

3. Evaluate $\lim_{x\to1}\frac{1-\sqrt{x}}{x\sin\pi x}=?$

(A)$-\frac{2}{\pi}$ (B)0 (C)$\frac{2}{\pi}$ (D)$\frac{1}{2\pi}$ (E) does not exist

4. Evaluate $\frac{d}{dx}\left[\log_{\sqrt{5}}\sec^{-1}(1+x)\right]$=?

(A)$\frac{2}{\ln 5\cdot\sec^{-1}(1+x)\cdot\sqrt{1-(1+x)^2}}$ (B)$\frac{2}{\ln 5\cdot\sec^{-1}(1+x)+\sqrt{1-(1+x)^2}}$ (C)$\frac{5}{\ln 5\cdot\sec^{-1}(1+x)\cdot\sqrt{1-(1+x)^2}}$

(D)$\frac{2}{\ln 5\cdot\sec^{-1}(1-x)\cdot\sqrt{1-(1+x)^2}}$ (E)$\frac{\sqrt{5}}{\ln 5\cdot\sec^{-1}(1+x)\cdot\sqrt{1-(1+x)^2}}$

5. If $y=2^{\tan^{-1}x}+e^{x^{\sqrt{x}}}$, please find $\frac{dy}{dx}$=?

(A)$\frac{\ln 2\times 2^{\tan^{-1}x}}{1+x^2}+x^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln x+\sqrt{x}\frac{1}{x}\right]$ (B)$\frac{\ln 2\times 2^{\tan^{-1}x}}{1-x^2}+x^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln x+\sqrt{x}\frac{1}{x}\right]\cdot e^{x^{\sqrt{x}}}$

(C)$\frac{\ln 2\times 2^{\tan^{-1}x}}{1+x^2}+x^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln x+\sqrt{x}\frac{1}{x}\right]\cdot e^{x^{\sqrt{x}}}$ (D)$\frac{\ln 2\times 2^{\tan^{-1}x}}{1-x^2}+x^{\sqrt{x}}\left[\frac{1}{\sqrt{x}}\ln x+\sqrt{x}\frac{1}{x}\right]\cdot e^{x^{\sqrt{x}}}$

(E) $\frac{\ln 2\times 2^{\tan^{-1}x}}{1-x}+x^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln x+\sqrt{x}\frac{1}{x}\right]\cdot e^{x^{\sqrt{x}}}$

6. Considering a function $\sin(xy)=4y\arccos(x-1)$, please find the equation of normal line at point $(1,2\pi)$.

(A) $y-2\pi=\frac{2\pi-1}{10\pi}(x-1)$ (B) $y-2\pi=\frac{10\pi}{1-2\pi}(x-1)$ (C) $y-2\pi=\frac{10\pi}{2\pi-1}(x-1)$

(D) $y-2\pi=\frac{1-2\pi}{10\pi}(x-1)$ (E) $y-2\pi=\frac{5\pi}{2\pi-1}(x-1)$

7. If $f(x)=1+x+e^{-x}$, $f(0)=2$ please evaluate $\left.\frac{d(f^{-1})}{dx}\right|_{x=0}$ =?

(A)0 (B)$\frac{1}{3}$ (C)$\frac{1}{2}$ (D)1 (E)Does not exist.

8. If $f(x)$ is continuous on [0 , 2] and differentiable in (0 , 2). Suppose that $f(0)=-3$ and $1<f'(x)<2$ for all x in (0 , 2). Find a possible value of $f(2)$.

(A)2 (B)3 (C)4 (D)5 (E) 6

9. Evaluate $\frac{d}{dx}\int_{3x+x^2}^{0} t\sqrt{(1+t^2)^5}\,dt$ =?

(A) $-(3x+x^2)\sqrt{\left[1+(3x+x^2)^2\right]^5}\cdot(3x+x^2)$ (B) $-(3x+x^2)\sqrt{\left[1+(3x+x^2)^2\right]^5}\cdot(3+2x)$

(C) $-(3x+x^2)\sqrt{\left[1+(3x+x^2)^2\right]}\cdot(3+2x)$ (D) $(3x+x^2)\sqrt{\left[1+(3x+x^2)^2\right]^5}\cdot(3+2x)$

(E) $(3x+x^2)\sqrt{\left[1+(3x+x^2)^2\right]^5}\cdot(3x+x^2)$

10.Evaluate $\int_{-10}^{10}\frac{\sin 6x}{\sqrt{1+x^{100}}}dx=?$

(A)-2 (B)-1 (C)0 (D)1 (E) 2

11. If $f(x)$ is a continuous function defined in R, and $\int_0^{x^2} f(t)dt = x\,arc\sec\pi x$, please find $f(9)$.

(A) $\frac{3\pi}{\sqrt{1-(3\pi)^2}}-1$ (B) $\frac{\pi}{2\sqrt{1-(3\pi)^2}}-\frac{1}{6}$ (C) $\frac{\pi}{2\sqrt{1-(3\pi)^2}}+\frac{1}{6}$ (D) $\frac{3\pi}{\sqrt{1-(3\pi)^2}}+1$ (E) $\frac{\pi}{\sqrt{1-(3\pi)^2}}-\frac{1}{6}$

12. Evaluate $\int \arccos\sqrt{x}\,dx$ =?

(A) $\cos^{-1}\sqrt{x}+2\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$ (B) $x\cos^{-1}\sqrt{x}-2\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$

(C) $x\cos^{-1}\sqrt{x}+2\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$ (D) $x\cos^{-1}\sqrt{x}+\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$

(E) $x\cos^{-1}\sqrt{x}+2\sqrt{1-x}+\frac{2}{3}(1-x)^{\frac{3}{2}}$

13. Evaluate $\int \tan^4 x\,dx$ =?

(A) $(\sec^2 x-1)^2+c$ (B) $\frac{1}{3}\tan^3 x+\tan x+x+c$ (C) $\frac{1}{3}\tan^3 x-\tan x-x+c$

(D) $(\sec^2 x+1)^2+c$ (E) $\frac{1}{3}\tan^3 x-\tan x+x+c$

14. Evaluate $\int\frac{1}{1+\sin x}dx=?$

(A) $\tan x+C$ (B) $\ln|1+\sin x|+C$ (C) $\tan x+\sec x+C$ (D) $\tan x-\sec x+C$ (E) $\sec x+C$

15. Evaluate $\int \frac{1}{3+\cos x-\sin x}dx = ?$(hint: let $tan\frac{x}{2} = t$)

(A) $\frac{\sqrt{7}}{2}\tan^{-1}\left[\frac{\sqrt{7}}{2}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$ (B) $\frac{\sqrt{7}}{2}\tan^{-1}\left[\frac{2}{\sqrt{7}}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$ (C) $\frac{2}{\sqrt{7}}\tan^{-1}\left[\frac{2}{\sqrt{7}}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$

(D) $\frac{2}{\sqrt{7}}\tan^{-1}\left[\frac{\sqrt{7}}{2}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$ (E) $\frac{4}{\sqrt{7}}\tan^{-1}\left[\frac{\sqrt{7}}{2}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$

16. Find the volume of the solid of the revolution formed by revolving the region bounded by $y = 1-\frac{x^2}{16}$and the x-axis $0 \le x \le 4$ about the x-axis.

(A) $\frac{32}{15}\pi$ (B) 2π (C) $\frac{28}{15}\pi$ (D) $\frac{26\pi}{15}$ (E) $\frac{8}{5}\pi$

17. Evaluate $\int \frac{2x^3-4x^2-15x+5}{x^2-2x-8}\,dx = ?$

(A) $x^2+\frac{3}{2}\ln|x-4|-\ln|x+2|+c$ (B) $x^2+\frac{3}{2}\ln|x-2|-\frac{1}{2}\ln|x+4|+c$

(C) $x^2+\ln|x-4|-\frac{1}{2}\ln|x+2|+c$ (D) $x^2+\frac{3}{2}\ln|x-4|+\ln|x+2|+c$

(E) $x^2+\frac{3}{2}\ln|x-4|-\frac{1}{2}\ln|x+2|+c$

18. Evaluate $\int \frac{1}{a^4-x^4}dx = ?$

(A) $\frac{1}{2a^3}\left[\ln|a+x|+\ln|a-x|+2\tan^{-1}\frac{x}{a}\right]+c$ (B) $\frac{1}{4a^3}\left[\ln|a+x|-\ln|a-x|+2\tan^{-1}\frac{x}{a}\right]+c$

(C) $\frac{3}{4a^3}\left[\ln|a+x|-\ln|a-x|+\frac{1}{a}\tan^{-1}\frac{x}{a}\right]+c$ (D) $\frac{x}{4a^3}\left[\ln|a+x|-\ln|a-x|+\frac{x}{a}\tan^{-1}\frac{x}{a}\right]+c$

(E) $\frac{1}{4a^3}\left[\ln|2a+x|+\ln|a-x|+\tan^{-1}\frac{x}{a}\right]+c$

19. Evaluate $\int_1^7 \frac{\ln x}{x^3}dx = ?$

(A) $\frac{1872}{625}$ (B) $\frac{-1}{3}\left[\frac{2\ln 5+1}{50}-\frac{1}{2}\right]$ (C) $\frac{24-2\ln 7}{100}$ (D) $\frac{-1}{2}\left[\frac{2\ln 7+1}{50}-\frac{1}{4}\right]$ (E) $\frac{1}{2}\left[\frac{2\ln 7+1}{50}-\frac{1}{2}\right]$

20.Evaluate $\lim_{x\to 4}\frac{x\int_4^x \frac{\cos t}{t}dt}{x-4} = ?$

(A) $\sin 4$ (B) $\sin 8$ (C) $\cos 8$ (D) $\cos 4$ (E) $\tan 8$

第二部分:非選題共一題，計 20 分，請將答案用藍黑色原子筆寫在答案紙上。

1. Assume that $|\varepsilon| << 1$, determine a, b, a', b' for the solutions of this function $x^2 + (\varepsilon - 4)x + (3 + 2\varepsilon) = 0$;

$x^{(1)} \cong 1 + a\varepsilon + b\varepsilon^2 + O(\varepsilon^3)$
$x^{(2)} \cong 1 + a'\varepsilon + b'\varepsilon^2 + O'(\varepsilon^3)$

$x^{(1)}, x^{(2)}$ are the solutions of this function, where the remainder terms $O(\varepsilon^3), O'(\varepsilon^3)$ are postulated to be significantly less than prior terms.