國立宜蘭大學

103 學年度第一學期微積分競試試題

※注意事項※

1. 考試時間為 100 分鐘，考試開始 10 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機、及任何通訊器材(參考數值請參照 Appendix)。
2. 試題共兩部分，第一部份為選擇題，每題 4 分，共 80 分。第二部分為非選擇題，共 20 分。試題答案請依題號填入答案卡,答錯或劃記多於一個選項者倒扣 1 分，倒扣到總分數零分為止,未作答者,不給分亦不倒扣。
3. 請用 2B 鉛筆在答案卡之「解答欄」內劃記。修正時應以橡皮擦拭,請勿在答案卡上使用修正液。作答範例：若第 1 題試題選項為(A)3 (B)5 (C)7 (D)9 (E)11,而正確的答案為選項(A)3 時，請在答案卷上劃記 (請實心填滿或大部分填滿) 。

祝考試順利!!

第一部分：選擇題，每題4分，共80分。請將答案填在答案卡上。

1. Assume $f(x,y)=\ln\sqrt[4]{x^2+y^2}$ ，please find $f_{xx}+f_{yy}=?$

(A)0 (B)$\left(x^2+y^2\right)^{-2}$ (C)$2\left(x^2+y^2\right)^{-2}$ (D)$\frac{1}{2}\left(x^2+y^2\right)^{-2}$

2. Assume $f(x)=\frac{x(x-1)(x-2)(x-3)....(x-n)}{(x+1)(x+2)(x+3)....(x+n)}$ ，please find $f'(0)=?$

(A)0 (B)1 (C)-1 (D)$(-1)^n$

3. For what interval of ***x*** does the series $\sum_{k=1}^{\infty}\frac{7^k}{k!}x^k$ converge?

(A)(-1,1) (B)(-7,7) (C)(-∞,∞) (D) not exist

4. $\int_0^3\frac{1}{(1+x)\sqrt{x}}dx=?$

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π

5. $\int_0^{\frac{\pi}{2}}\frac{1}{\sin x+\cos x}dx=?$

(A)$\sqrt{2}\ln\left(\sqrt{2}+1\right)$ (B)$\sqrt{2}\ln\left(\sqrt{2}-1\right)$

(C)$\sqrt{2}\ln\left(\sqrt{2}+1\right)^2$ (D) $\frac{1}{\sqrt{2}}\ln\left(\sqrt{2}+1\right)$

6. Let $f:R\to R$ be a twice continuously differentiable function,

$f(0)=1, f(1)=2\ and\ f'(1)=0$ .Compute $\int_0^1 xf''(x)dx$.

(A)-1 (B)0 (C)1 (D)2

7. Evaluate $\int_0^1\int_{\sqrt{x}}^1\sin y^3dydx$.

(A) $\frac{1}{3}\cos 1-\frac{1}{3}$ (B)$\frac{1}{3}\sin 1-\frac{1}{3}$ (C)$-\frac{1}{3}\cos 1+\frac{1}{3}$ (D)$-\frac{1}{3}\sin 1+\frac{1}{3}$

8. Assume $f(x,y)=\int_{x}^{y^2} e^{2t}dt+\int_{x^3}^{y}\cos t^2 dt=0$ ，please find $\dfrac{dy}{dx}$.

(A) $\dfrac{e^{2x}+3x^2\cos x^6}{ye^{2y}+\cos y^2}$ (B) $\dfrac{e^{2x}-3x^2\cos x^6}{2ye^{2y}+\cos y^2}$

(C) $\dfrac{-e^{2x}-3x^2\cos x^6}{2ye^{2y}+\cos y^2}$ (D) $\dfrac{e^{2x}+3x^2\cos x^6}{2ye^{2y}+\cos y^2}$

9. $\int_0^1\int_0^{\sqrt{1-x^2}}\left(x^2+y^2\right)^{-\frac{1}{2}}dxdy=?$

(A) π (B) $\dfrac{\pi}{2}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{4}$

10. Evaluate $\sum_{n=1}^{\infty}\ln\left[\dfrac{(n+1)^2}{n(n+2)}\right]$.

(A) Not exist (B)0 (C) $\ln 2$ (D) $\ln 3$

11. Evaluate $\int \sin^2 x\cot^3 x dx$.

(A) $\ln\cos x-\frac{1}{2}\cos^2 x+C$ (B) $\ln\sin x-\frac{1}{2}\sin^2 x+C$

(C) $\ln\cos x-\frac{1}{2}\sin^2 x+C$ (D) $\ln\sin x-\frac{1}{2}\cos^2 x+C$

12. Find $\lim_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}}$.

(A) 1 (B) $e^{\frac{1}{2}}$ (C) $e^{\frac{1}{3}}$ (D) $e^{\frac{1}{4}}$

13. $\int_4^5\dfrac{3x-7}{(x-1)(x-2)(x-3)}dx=?$

(A) $4\ln 3+4\ln 2$ (B) $3\ln 3+4\ln 2$

(C) $4\ln 3-4\ln 2$ (D) $3\ln 3-4\ln 2$

14. Consider $f(x,y,z)=x^3y+2y^2z^2+x^2z^3$ at the point (2,1,-1) .Find the maximum rate of change of $f(x,y,z)$.

(A) $4\sqrt{17}$ (B) $4\sqrt{21}$ (C) $4\sqrt{37}$ (D) $4\sqrt{59}$

15. Assume $f(\frac{1+x}{1-x})=x$ ， please find $f'(x)=?$

(A) $\frac{1+x}{1-x}$ (B) $\frac{1}{x}$ (C) $\frac{2}{(1+x)^2}$ (D) 1

16. Which of the following series is divergent?

(A) $\sum_{n=1}^{\infty}\sin\frac{1}{n}$ (B) $\sum_{n=1}^{\infty}\frac{\tan^{-1}n}{n^2+1}$ (C) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$ (D) $\sum_{n=1}^{\infty}\frac{1}{n(\ln n)^s}$ $(s>1)$

17. The region enclosed by $y=e^{-x}$, $x=0$ and $x=1$ is rotated about

the line $y=0$. Find the volume of the resulting solid.

(A) $\frac{\pi(e^2-1)}{e^2}$ (B) $\frac{2\pi(e^2-1)}{3e^2}$ (C) $\frac{\pi(e^2+1)}{e^2}$ (D) $\frac{\pi(e^2-1)}{2e^2}$

18. If a$\neq$0, $\lim_{x\to 0}\left(\frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}\right)$

(A) –sina (B) 0 (C) sin(2a) (D)2sina

19. Let $f(x,y)=x^{2^y}$, find $\frac{\partial f}{\partial y}$.

(A) $x^{2^y}\cdot 2\ln x\cdot 2^y\cdot\ln 2$ (B) $x^{2^y}\cdot\ln x\cdot y\cdot 2^{y-1}$

(C) $x^{2^y}\cdot 2^y\cdot\ln 2\cdot\ln x$ (D) $x^{2^y}\cdot\ln x\cdot 2^{y-1}$

20. Suppose that $f(x)=\begin{cases}3k+\sqrt{x} & 0\le x\le 4\\ 2kx-7x & 4<x\le 9\end{cases}$ is continuous on [0,9]. Then

k=?

(A)6 (B)7 (C)8 (D)9

第二部分:非選題，20分，請將答案用藍黑色原子筆寫在紙上。

Assume human population size (人口數量大小) x at time t can be presented as $x = \frac{M}{1+\exp(-\alpha(t-t_0))}$, where α and the maximal population size M are positive constants and t_0 is the time at which $x=\frac{1}{2}M$.

Suppose a census (人口調查) is taken at <u>three equally spaced time t_1, t_2 and t_3</u>, the resulting numbers being x_1, x_2 and x_3. Show that

$$M = x_2 \frac{x_3(x_2 - x_1) - x_1(x_3 - x_2)}{x_2^{\,2} - x_1 x_3}.$$