國立宜蘭大學 107 年度微積分競試 試題

※注意事項※

1. 考試時間為 100 分鐘(13:10-14:50)，考試開始 10 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機及任何通訊器材。

2. 本試題共計 24 題，總分為 112.8 分。

3. 各題答案請依題號填入答案卷上相對應題號的空格內，填錯格或填在格外者不予計分，字跡切勿潦草，答錯或未作答者，不給分亦不倒扣。

4. 請將您的班級、學號及姓名，用正楷填寫於答案卷上方的欄位內。

5. 考試結束時，請將答案卷繳回即可，本試題不必繳回。

6. 14:00 後才能提早交卷。

祝金榜題名!!!

1. If $f(x) = \dfrac{x}{3x-1}$, $g(x) = \sqrt{4-x}$. Find the domain of $g \circ f(x)$.

2. Solve the equation $e^{2x+1} = 3^{x-5}$.

3. Evaluate $\lim\limits_{x \to 1} \dfrac{x^2-1}{|x-1|}$, if it exists.

4. Suppose that the diameter of a cat's pupils is given by

   $f(x) = \dfrac{21x^{-0.2}+8}{3x^{-0.2}+2}$ (mm), where x is the intensity of light on the

   pupils. Find the diameter A with minimum light and the diameter B with maximum light. (A, B) = _____

5. $\lim\limits_{(x,y) \to (1,2)} \dfrac{5x^2 y}{x^2+y^2}$ = _____

6. Calculate $\dfrac{dy}{dx}$ using implicit differentiation. $4y^2 + \ln(x^2 y) = 7$

7. Find the derivative of the function $y = \dfrac{x(x-1)^{\frac{3}{2}}}{\sqrt{x+1}}$, $x > 1$.

   (答案限用 x 表示)

8. Find the point on the graph of the function $f(x) = \sqrt{x}$ that is closest to the point $(2, 0)$.

9. Discuss the concavity of the graph of the function $f(x) = (x-2)^3(x-1)$

10. Find the extrema of the given function $f(x) = x^2 e^{-x}$

11. $\int \dfrac{(\ln x)^2}{x} dx$ = _____

12. $\int \sec^2 \dfrac{x}{2} \tan \dfrac{x}{2} dx$ = _____

13. $\int_0^{\frac{3}{5}} \sqrt{9 - 25x^2} \; dx$ = _____

14. $\int_1^2 \dfrac{x+1}{x(x^2+1)} dx$ = _____

15. Find the volume of the solid formed by revolving the region bounded by
    $y = \dfrac{1}{3}x^3$ and $y = 6x - x^2$ about the line $x = 3$.

16. Find the divergence at $(2,1,1)$ for the vector field
    $F(x, y, z) = xyz\mathbf{i} + xy\mathbf{j} + z\mathbf{k}$.

17. Find the maximum value of the directional derivative of the function
    $f(x, y) = x \tan y$ at the given point $(2, \dfrac{\pi}{4})$.

18. Find the maximum value of $f(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the

constraint $2x - 3y - 4z = 49$.

19. Let R be the annular region lying between the two circles $x^2 + y^2 = 1$

and $x^2 + y^2 = 5$. Evaluate the integral $\iint\limits_{R} (x^2 + y)dA$, where A is the

area of R.

20. Find the equation of the tangent plane to the hyperboloid

$z^2 - 2x^2 - 2y^2 = 12$ at the point $(1, -1, 4)$.

21. 已知 $x = s^2 - t^2$, $y = \dfrac{s}{t}$ 和 $w = 2xy$, 求 $\dfrac{\partial w}{\partial t}$ (答案限用 s 及 t 表示)

22. 已知 $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0$, 求 $\dfrac{\partial z}{\partial y}$

23. 求以拋物面 $z = 4 - x^2 - 2y^2$ 和 xy平面 為界的立體區域體積

24. 求圖形 $r = 3\cos 3\theta$ 所圍出的面積