

# 國立宜蘭大學 109 年度微積分競試 試題

## ※注意事項※

1. 考試時間為 100 分鐘(13:10-14:50)，考試開始 10 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機及任何通訊器材。
2. 本試題共計 22 題，總分為 102.6 分。
3. 各題答案請依題號填入答案卷上相對應題號的空格內，填錯格或填在格外者不予計分，字跡切勿潦草，答錯或未作答者，不給分亦不倒扣。
4. 請將您的班級、學號及姓名，用正楷填寫於答案卷上方的欄位內。
5. 考試結束時，請將答案卷繳回即可，本試題不必繳回。
6. 14:00 後才能提早交卷。

祝金榜題名!!!

**1-8 題每題 4 分**

1. Test the series for convergence or divergence.  $\sum_{n=1}^{\infty} (-1)^n \frac{n^3 - 1}{n^4 + 1}$

2. Test the series for convergence or divergence.  $\sum_{n=1}^{\infty} \frac{n^7 - 1}{n^8 + 1}$

3. If  $f^{(n)}(0) = (n+1)!$  for  $n = 0, 1, 2, \dots$ , find the Maclaurin series for  $f$ .

4. Find the gradient vector field of  $f$ .  $f(x, y, z) = x \cos\left(\frac{3y}{z}\right)$

5.  $f(x)$  為連續函數時的  $c$  值。  $f(x) = \begin{cases} cx^2 - 3, & x \leq 2 \\ cx + 2, & x > 2 \end{cases}$

Find the limit of the following function if it exists. Otherwise, fill the blank with “X”, if the limit does not exist.

6.  $\lim_{x \rightarrow 3^-} \frac{x^2 + x + 2}{x^2 - 2x - 3}$

7.  $\lim_{x \rightarrow \infty} \frac{3x^2}{x^3 - x^2 + x}$

8.  $g(x) = \begin{cases} |x|, & \text{if } x \neq 0 \\ 5, & \text{if } x = 0 \end{cases}$ , find  $\lim_{x \rightarrow 0} g(x)$ .

**9-16 題每題 5 分**

9. Find the equations of all lines having a slope of 6 and being tangent to the curve of the following equation: (in the form of  $y = a x + b$ ).

$$y + \frac{3}{2x-5} = 0$$

10. Approximate  $\sqrt[3]{125.3}$  to five decimal places.

11. Find  $y'$  by implicit differentiation from the following equation:

$$\tan(x^2 y^4) = 3x + y^2$$

12. Suppose that  $f(x)$  is continuous on the closed interval  $[-7, 0]$  and differentiable on the open interval  $(-7, 0)$ . In addition, if  $f(-5) = -3$  and  $f'(x) \leq 2$  for all  $x$  in the open interval  $(-7, 0)$ , what is the largest possible value for  $f(-1)$ ?

13. Evaluate  $\int_1^4 (3 - |x-3|) dx$

14. Evaluate  $\int \frac{x}{\sqrt{x^2 + 6x + 12}} dx$

15. Evaluate  $\int t \ln(t+1) dt$

16. Evaluate  $\int_0^{\pi/2} e^{\sin \pi x} \cos \pi x dx$

**17-22 題每題 5.1 分**

17. Find the arc length of the curve.  $y = \ln(\sec x)$ ,  $0 \leq x \leq \pi/4$

18. Find  $\frac{\partial z}{\partial x}$  of the equation  $x + y^y = \sin(xy + z)$

19. Find all the relative extrema of the following:

$$f(x) = 2x - \frac{4}{3} \arctan 3x$$

20.  $\int_0^4 \int_0^{x^2} x \cos(y) dy dx$

21.  $\int_0^{\frac{1}{\sqrt{2}}} \int_y^{\sqrt{1-y^2}} e^{6x^2+6y^2} dx dy$

22. The function  $f(x, y) = ax + by$  has an average value of 25 on the rectangle  $0 \leq x \leq 4, 0 \leq y \leq 2$ . What can you say about the constants  $a$  and  $b$ ? (Give your answer as an equation that describes the values of  $a$  and  $b$ )