國立宜蘭大學 112 年度第二次微積分競試 試題

※注意事項※

1. 考試時間為 100 分鐘(13:10-14:50)，考試開始 20 分鐘後不得入場，考試期間不得離開考場；考試期間亦禁止使用字典、計算機及任何通訊器材。
2. 本試題共計 22 題，總分為 102.6 分。
3. 各題答案請依題號填入答案卷上相對應題號的空格內，填錯格或填在格外者不予計分，字跡切勿潦草，答錯或未作答者，不給分亦不倒扣。
4. 請將您的班級、學號及姓名，用正楷填寫於答案卷上方的欄位內。
5. 考試結束時，請將答案卷繳回即可，本試題不必繳回。
6. 14:00 後才能提早交卷。

祝金榜題名!!!

<u>1-8 題每題 4 分</u>

1. 設 (x, y) 為 $y=\sqrt{x-3}$ 圖形上的一點。令 L 表示 (x, y) 至 $(4,0)$ 的距離。請將 L 表為 y 的函數。

2. 求作函數 $y=|x^2-1|$ 的圖形。

3. 求有理函數 $h(x)=\dfrac{7x^3}{x^3-3x^2+6x}$ 的極限值：當 $x\to\infty$ 。

4. 若 $\lim_{x\to 4}\dfrac{f(x)-5}{x-2}=1$ ，求極限值 $\lim_{x\to 4} f(x)$ 。

5. Test the series for convergence or divergence: $\sum_{n=1}^{\infty}\frac{n^2-1}{n^3+1}$

6. Test the series for convergence or divergence: $\sum_{n=1}^{\infty}\frac{e^n}{n^2}$

7. Find the equation of the tangent plane to the surface: $x^2+z^2e^{(y-x)}=5$ at the point (1, 1, 2). Give your answer in the form of Ax + By + Cz = D.

8. 承上題，find the parametric equation of the normal line to the tangent plane.

<u>9-16 題每題 5 分</u>

9. $y = \frac{x^3}{\cos x}$. Compute the derivative y'.

10. $y = e^{-\frac{x^2}{2}}$. Evaluate the second derivative y''.

11. Determine the derivative for the function $y = x^{x-1}$. $y' =$?

12. Find the minimum value of the function $f(x) = x - 2\sin x,\ x \in [0, \pi]$.

The minimum is equal to?

13. Find the slope m for the equation $(4 - x)y^2 = x^3$ at the point $(2,2)$.

The slope $m =$?

14. Evaluate the integral $\int x^2\sqrt{x^3 + 1}\,dx$

15. Evaluate the integral $\int_2^3 \frac{x(3-5x)}{(3x-1)(x-1)^2}dx$

16. Evaluate the integral $\int_0^1 \int_{3y}^3 e^{x^2}\,dxdy$

17-22 題每個答案 5.1 分

17. Evaluate the gradient at the point P. $f(x, y) = x^2 e^y$, $P(3,0)$

18. Evaluate the integral to calculate the area of the region bounded by two lines, $3y = 2x + 16$ and $y = -2x + 8$, and a curve $y = x^2$.

19. Find the volume obtained by rotating the region, bounded by the given curves $x = 2y^2$, $x = 2$, $y \geq 0$, about the $y = 2$ axis.

20. The surfaces, $f(x, y, z) = 3x^2+y^2+2z^2-9 = 0$ and $g(x, y, z) = x^2+2y^2+ z^2-10 = 0$, meet in a curve C. Find the parametric equations for the line tangent to C at the point P(1, 2, 1).

21. Find the point P farthest to the origin on the curve of intersection of the plane $x + y + z = 3$ and the sphere $(x-1)^2 + (y-2)^2 + (z-3)^2 = 9$.

22. Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where

$$\vec{F}(x, y, z) = \mathbf{e}^{2x}\,\hat{\imath} + z\,(y+1)\,\hat{\jmath} + z^3\,\hat{k}$$

and C is given by

$$\vec{r}(t) = t^3\,\hat{\imath} + (1-3t)\,\hat{\jmath} + \mathbf{e}^t\,\hat{k}, \text{ for } 0 \leq t \leq 2.$$